On the distribution and correlation of the sexes (staminate and pistillate flowers) in the inflorescence of the aroids Arisarum vulgare and Arisarum proboscideum

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(with two text figures)

I. Introductory Remarks

Because of its high degree of specialization the inflorescence of the aroids should present most interesting, and in some instances favorable, material for biometric studies.

So far as I am aware, however, practically nothing is known in quantitative terms of variation and correlation in the inflorescence of this group. In this note I present the results of an analysis of the data for two species collected by Cannarella. I hope that the publication of these results will suggest to those who have the opportunity, the desirability of obtaining more adequate series of quantitative data on this group. The structural peculiarities of the Aroids are too well known to botanists to require comment. Those who wish further details for *Arisarum* may consult the well-known monographs by Engler. The two species here considered are figured in Curtis's Botanical Magazine, *pl. 6023*, *6634*.

In the case of *Arisarum vulgare* Cannarella* has given data for number of male and female flowers in the inflorescence. These he tables in several different ways, but without bringing out the full significance of the observations. For *Arisarum proboscideum*† he has recorded the data for number of staminate and pistillate flowers per inflorescence for a series of individual habitats. Beyond a descriptive discussion he gives no analyses of these data except to state that the correlation between the number of male and female flowers for the total material is $r = -.069$. A negative value in a character such as the present leads one to suspect arith-

* Cannarella, P. Ricerche intorno di limite di variabilitá dell' *Arisarum vulgare* Torg. Nuovo Gior. Bot. Ital. N. S. 12: 328–347. 1905.

† Cannarella, P. Ricerche sull' apparato florale dell' *Arisarum proboscideum* Savi. Cont. Biol. Veg. R. Ist. Bot. Palermo 4: 121–138. 1909.

metical error. As will be shown presently the correlation is not negative but positive in sign, though numerically small. For all details concerning habitats and other biological conditions the original papers must be consulted.

II. STATISTICAL ANALYSIS OF DATA

Consider first the means and variabilities of the number of staminate and pistillate flowers in the inflorescences of the two species. For this discussion only the total materials collected for each species will be used. In *A. vulgare* the range of variation in number of staminate flowers is very great. The classes and the frequency with which they occur, together with the total number of pistillate flowers found on the inflorescences, are shown in TABLE I.

TABLE I

FREQUENCY OF STAMINATE FLOWERS AND ASSOCIATED PISTILLATE FLOWERS IN *A. vulgare*

Staminate flowers	*f*	Total pistillate flowers	Staminate flowers	*f*	Total pistillate flowers
8	1	3	35	41	209
14	1	6	36	37	202
16	1	13	37	37	254
17	1	1	38	30	189
18	3	12	39	39	263
19	2	15	40	21	140
20	1	4	41	26	175
21	4	13	42	21	147
22	3	14	43	11	74
23	8	37	44	14	108
24	13	53	45	6	50
25	9	50	46	5	38
26	23	119	47	2	14
27	14	64	48	4	39
28	16	80	49	3	31
29	31	132	50	2	19
30	33	155	51	1	5
31	30	156	53	1	6
32	37	186	57	3	42
33	33	164	59	1	7
34	50	269	Totals.........	619	3,558

The variation in the number of pistillate flowers may be calculated from the totals of TABLE II. The data for *A. proboscideum* appear in the totals of TABLE III. From these I deduce the constants for number of flowers per inflorescence given in TABLE IV.

The fact that the two species differ greatly in the numbers of the two types of flowers which they produce is well known to taxon-

omists. Any discussion of the differences in the means of the two species would therefore be superfluous. Since the mean numbers are all conspicuously lower in *A. proboscideum* one would naturally expect the absolute variabilities as measured by the standard deviations to be lower also. The constants show that this is actually the case. The coefficients of variation show that *A. proboscideum* is also *relatively* far less variable than *A. vulgare*. The result is rather surprising in view of the fact that the collections of *A. proboscideum* were apparently made from a wider range of habitats than those for *A. vulgare*.

The coefficients of variation for number of pistillate flowers are in both species conspicuously higher than those for staminate flowers. The interpretation of this result presents considerable difficulty. Variation in both cases is so low that a deviation of a unit is relatively far greater than in the case of the staminate flowers. Thus without a full consideration of the statistical difficulties presented by the case one cannot assert that biologically there is a greater tendency to variability in the number of pistillate than in the number of staminate flowers.

TABLE II

CORRELATION IN INFLORESCENCES OF *A. vulgare*

PISTILLATE FLOWERS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Totals
Staminate flowers:																	
6–10	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
11–15	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
16–20	1	—	—	4	1	—	—	—	—	1	—	—	1	—	—	—	8
21–25	1	1	13	10	—	5	1	6	—	—	—	—	—	—	—	—	37
26–30	—	2	22	42	23	13	6	7	—	1	1	—	—	—	—	—	117
31–35	—	2	31	54	29	33	18	20	2	—	—	2	—	—	—	—	191
36–40	—	—	7	26	22	35	28	30	7	3	—	4	2	—	—	—	164
41–45	—	—	5	7	9	11	12	17	6	4	1	3	3	—	—	—	78
46–50	—	—	—	—	1	3	2	1	3	1	4	—	—	—	1	—	16
51–55	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
56–60	—	—	—	—	—	—	1	—	—	—	—	1	—	1	—	1	4
Totals.....	2	5	79	143	86	102	68	81	18	10	6	10	6	1	1	1	619

The correlation surfaces for the two species are given as TABLES II and III. That for *A. proboscideum* appears in units. For *A. vulgare* the range of variation in number of staminate flowers is so wide that it has seemed desirable to group in classes

of 5 units range. The correlation coefficient for the ungrouped material may be computed from the condensed correlation TABLE I. The correlation constants in TABLE IV are calculated from the ungrouped observations. The first point to be noted is that the less variable inflorescence of *A. proboscideum* shows a distinctly lower correlation between the number of staminate and pistillate flowers than does that of *A. vulgare*. The difference is .226 ± .025, a value 9 times as large as its probable error and so unquestionably significant.

TABLE III

CORRELATION IN INFLORESCENCE OF *A. proboscideum*

PISTILLATE FLOWERS

	1	2	3	4	5	6	Totals
Staminate flowers:							
10	—	1	—	—	—	—	1
11	1	—	1	—	—	—	2
12	3	10	—	—	—	—	13
13	5	38	7	—	—	1	51
14	10	105	19	2	2	—	138
15	8	144	81	18	15	—	266
16	8	195	186	55	16	—	460
17	13	304	416	34	14	—	781
18	8	302	218	26	12	—	566
19	3	76	106	29	25	2	241
20	—	49	58	26	20	—	153
21	—	20	36	12	7	—	75
22	—	6	31	6	1	—	44
23	—	6	3	—	1	—	10
24	—	—	1	—	—	—	1
Totals..................	59	1,256	1,163	208	113	3	2,802

If one expresses the results in terms of regression the linear equations are

For *A. vulgare*:

$$p = .3796 + .1560\,s,$$

$$s = 27.205 + 1.252\,p.$$

For *A. proboscideum:*

$$p = 1.0299 + .0955\,s,$$

$$s = 15.836 + .490\,p,$$

where s = number of staminate and p = number of pistillate flowers.

These values are deduced from the ungrouped observations.

For *A. vulgare* it seems desirable to calculate regression on the grouped values given in TABLE II. This gives for the correlation

$$r_{sp} = .440 \pm .022,$$

a value differing from that obtained from the ungrouped observations by only about one tenth its probable error. The regression equations are

$$s = 27.065 + 1.274\, p,$$
$$p = .5280 + .1518\, s.$$

These are the lines in the diagram, FIG. 1.

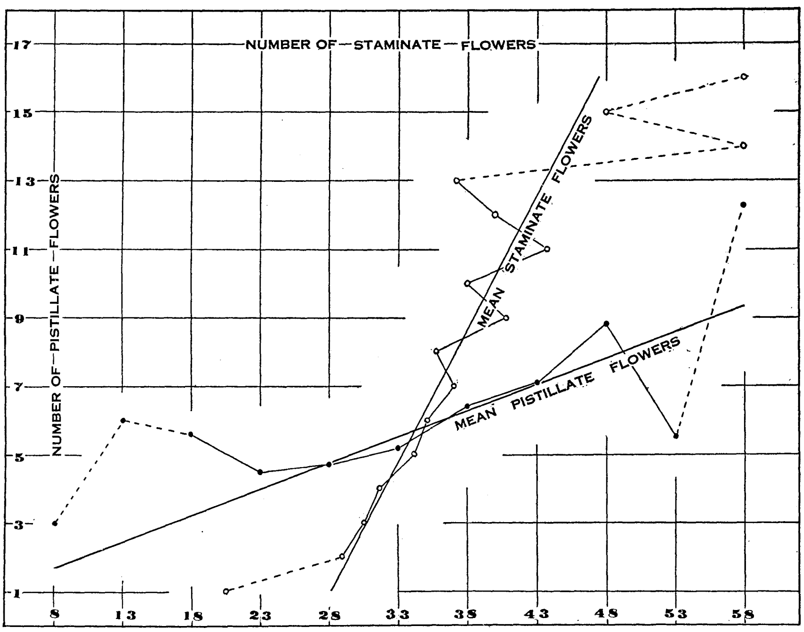

FIG. 1. Relationship between number of pistillate and number of staminate flowers in the inflorescence of *Arisarum vulgare*.—Empirical means and calculated lines.

These diagrams show considerable irregularities in the distribution of the means of the numbers of pistillate flowers associated with different numbers of staminate flowers and *vice versa*. It seems desirable, therefore, to apply a statistical test of the goodness of fit of the straight lines to the empirical means. Calculating

the correlation ratio* and using Blakeman's† well-known criterion

$$\frac{\zeta}{E\zeta} = \frac{\sqrt{N}}{.67449}, \quad \frac{1}{2}\sqrt{\zeta} \cdot \frac{1}{\sqrt{1 + (1 - \eta^2)^2 - (1 - r^2)^2}}.$$

I find:

	A. vulgare	*A. proboscideum*
Correlation coefficient	.440 ± .022	.216 ± .012
Correlation ratios:		
From means of pistillate flowers	.495 ± .020	.290 ± .012
Blakeman's criterion	4.38	7.87
From means of staminate flowers	.486 ± .021	.233 ± .012
Blakeman's criterion	3.96	3.37

TABLE IV

STATISTICAL CONSTANTS FOR *Arisarum*

	For *A. vulgare*	For *A. proboscideum*
Number of staminate flowers:		
Mean	34.402 ± .175	17.143 ± .024
S.D.	6.463 ± .124	1.850 ± .017
C.V.	18.79 ±0.37	10.79 ±0.10
Number of pistillate flowers:		
Mean	5.748 ± .062	2.668 ± .010
S.D.	2.282 ± .044	0.817 ± .007
C.V.	39.70 ±0.87	30.62 ±0.30
Total flowers per inflorescence:		
Mean	40.150 ± .210	19.810 ± .028
S.D.	7.747 ± .149	2.178 ± .020
C.V.	19.29 ±0.38	10.99 ±0.10
Correlation of staminate and pistillate flowers	.442 ± .022	.216 ± .012

The values of Blakeman's criterion all suggest significant non-linearity. The one which may be given the greatest weight is that for the regression of pistillate or staminate flowers in *A. proboscideum*. An examination of the diagram, FIG. 2, shows a peculiar and (on the basis of the data in hand) quite inexplicable periodicity in the distribution of the mean number of pistillate flowers. While the graphs themselves and the critical statistical constants indicate that straight lines do not exactly describe the interrelationship between the two variables, it is impossible to assert from the data as they stand that any curve of a higher order would express the rate of change in the mean value of an associated character

* Pearson, K. Drapers' Co. Res. Mem., Biom. Ser. II. 1905.

† Blakeman, J. Biometrika 4: 332–350. 1905.

better than is done by a straight line. Such deviations as do occur are probably due for the most part to the unavoidable errors of sampling. At any rate, the discussion of this point cannot be carried farther without more adequate biological information.

It is interesting to examine the results obtained for each of the sub-habitats from which *A. proboscideum* was collected.* These are the data combined to form our TABLE III.

The results for correlation between number of staminate and pistillate flowers, r_{sp}, are given in the third column of Table V. With but two exceptions the correlations are positive in sign. These two exceptional cases show very substantial negative coef-

TABLE V

CONSTANTS FOR INDIVIDUAL CORRELATIONS OF *A. proboscideum*

Locality	Number of plants	r_{sp}	r_{sp}/E_r	r_{ts}	r_{tz}	r_{tz}/E_r
Ospedallette......	341	$+.331\pm.033$	10.19	$+.922$	$-.413\pm.030$	13.68
Picarelli.........	490	$+.245\pm.029$	8.55	$+.876$	$-.482\pm.023$	20.59
Scierta..........	465	$+.090\pm.031$	2.89	$+.951$	$+.005\pm.031$	.16
Zigarelli.........	48	$+.290\pm.089$	3.25	$+.944$	$-.291\pm.089$	3.26
Contrada........	80	$+.276\pm.070$	3.96	$+.981$	$+.123\pm.074$	1.65
Ajello...........	13	$-.422\pm.154$	2.74	$+.952$	$+.399\pm.157$	2.53
Faliari..........	500	$+.187\pm.029$	6.42	$+.927$	$-.209\pm.029$	7.26
Lareto..........	685	$+.106\pm.026$	4.17	$+.915$	$-.181\pm.025$	7.27
Liquerini........	16	$-.566\pm.115$	4.94	$+.588$	$-.206\pm.161$	1.28
Montefredani.....	164	$+.077\pm.052$	1.47	$+.921$	$-.124\pm.052$	2.38

ficients, which are apparently significant in comparison with their probable errors. Since the number of individuals is so small—only 13 and 16—the conventional method of computing the probable error can not be given too much weight. The two exceptions to the rule cannot, therefore, be regarded as a real argument against the general existence of a positive correlation. The average value of these correlations, obtained by weighting with the numbers of individuals upon which the several coefficients are based, is $\bar{r}=+.170$. The difference between this value and the $r=+.216$ obtained from the whole (massed) material gives some idea of the influence of heterogeneity on the magnitude of the correlation.

Consider now the relationship between the total number of flowers produced per inflorescence and the relative numbers of the

* The numbers are in some cases far too small to give trustworthy constants, but it has seemed best to calculate the results for all alike to avoid the possible criticism of selection of data.

two types of flowers in the inflorescence. This seems a question of considerable interest because of the many discussions of the influence of feeding or degree of development on sex.

The problem is approached statistically as follows. Let z

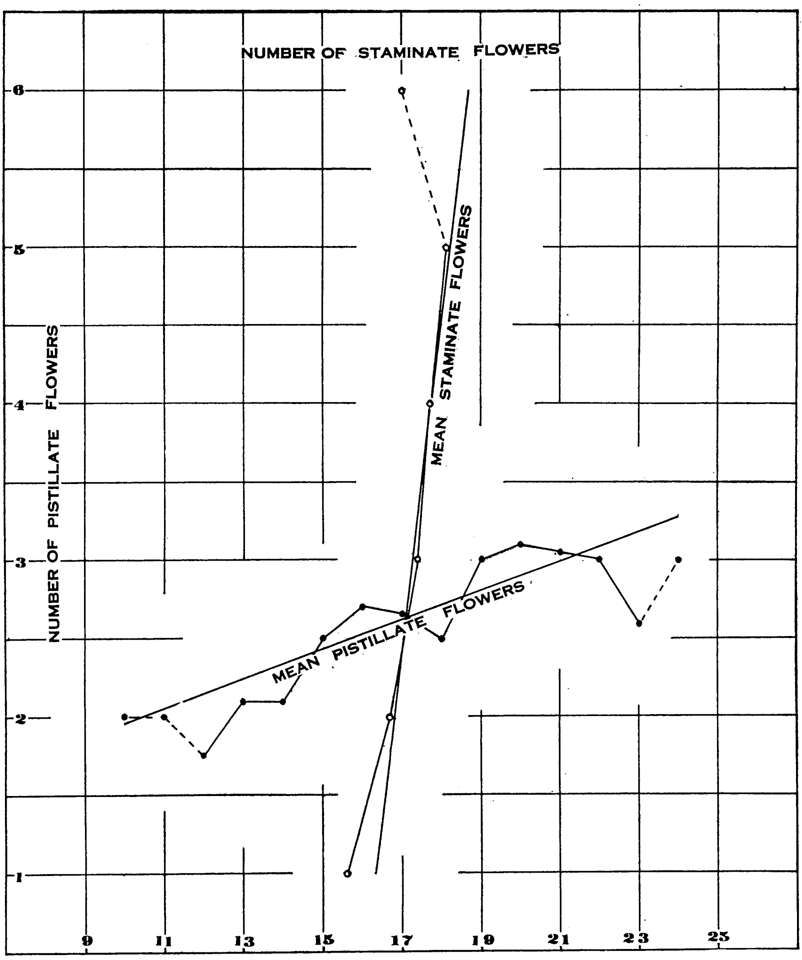

FIG. 2. Regression of number of staminate on number of pistillate, and of number of pistillate on number of staminate flowers in *A. proboscideum*.

represent the deviation of the number of staminate flowers per inflorescence from the probable number on the assumption that the proportion of the two types, s and p, is unaffected by the total number produced per inflorescence. If the correlation between

t, $= s + p$, or total flowers per inflorescence, and z be positive it indicates that as the total number of flowers per inflorescence becomes larger the staminate flowers are relatively more numerous. Using the formula suggested to me* by Professor Karl Pearson I find:

For *A. vulgare*:†

$$r_{tm} = + .964 \pm .002,$$

$$r_{tz} = - .230 \pm .026.$$

For *A. proboscideum:*

$$r_{tm} = + .931 \pm .002,$$

$$r_{tz} = - .234 \pm .012.$$

In both cases the correlation between total flowers and number of staminate flowers is high. This is as one would expect it to be. The interesting feature of the result is that in both species there is a significant *negative* correlation between the total number of flowers and the deviation of the number of staminate flowers from their probable value. In short, the male flowers while absolutely more abundant in the larger inflorescences are there *relatively* distinctly less abundant. Or in other words the larger inflorescences tend to produce a larger proportion of pistillate flowers.

Consider these relationships on the basis of the individual collections.

TABLE V, in which the constants for the individual collections of *A. proboscideum* are given, shows that with a single exception, in which N is very small, the correlations for total flowers and staminate flowers are very high.

The critical values derived from these are given in the sixth column. Of these 10 coefficients, 7 are negative in sign, and 5 may be considered unquestionably significant in comparison with their probable errors. Neither of the 3 positive values may be looked upon as certainly trustworthy. Thus while the constants for the individual sub-series are very irregular in magnitude they fully substantiate the conclusions to be drawn from the constant

* Harris, J. Arthur. The Correlation between a Variable and the Deviation of a Dependent Variable from its Probable Value. Biometrika **6**: 438–443. 1909.

† Calculations from ungrouped data.

for the whole material. The average (weighted) value is $\bar{r}_{tz} = -.223$. This value is not significantly different from the $r_{tz} = -.234$ obtained from the combined materials.

III. SUMMARY AND CONCLUSIONS

As a suggestion for more adequate investigation of variation and correlation in the specialized inflorescence of the aroids this note presents the results of a statistical analysis of two rather large series of data collected for the genus *Arisarum* by Cannarella.

The following points may be emphasized:

In both *A. vulgare* and *A. proboscideum*, the coefficient of variation for number of pistillate flowers is far higher than that for number of staminate flowers. The interpretation of this result presents considerable difficulty.

In *A. vulgare* the inflorescence not only produces a larger number of both staminate and pistillate flowers than does that of *A. proboscideum*, but shows a far higher variability, both absolute and relative.

Apparently the collections of *A. proboscideum* came from a wider range of habitats than those of *A. vulgare*. If this be true, the variability which they show is in part due to heterogeneity arising through environmental differentiation. This probability renders more significant the observed lower variability in *A. proboscideum*. Any subsequent investigation should if possible take into account this factor in drawing comparisons between the variability of these or any other forms.

The inflorescence of *A. vulgare* is not only more variable, but shows a higher degree of correlation of its parts than does that of *A. proboscideum*. In both cases the correlation between number of staminate and pistillate flowers is rather small. The relationship between the numbers of the two types of flowers may be fairly well described by a straight line. The fit of the straight line equations is not altogether satisfactory, but it seems idle to carry the statistical analysis farther on the present data. The effort would much better be expended in securing more adequate biological material.

The correlation between the total number of flowers produced by the inflorescence and the number of staminate flowers is neces-

sarily positive and high. The correlation between the total number of flowers and the deviation of the staminate flowers from their probable value, on the assumption that the character (sex) of the flowers is independent of the size of the inflorescence, is significantly negative. This shows that the larger inflorescences have relatively more pistillate flowers.

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